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Cascading failures and the emergence of cooperation in evolutionary-game based models of social and economical networks

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We study catastrophic behaviors in large networked systems in the paradigm of evolutionary games by incorporating a realistic "death" or "bankruptcy" mechanism. We find that a cascading bankruptcy process can arise when defection strategies exist and individuals are vulnerable to deficit. Strikingly, we observe that, after the catastrophic cascading process terminates, cooperators are the sole survivors, regardless of the game types and of the connection patterns among individuals as determined by the topology of the underlying network. It is necessary that individuals cooperate with each other to survive the catastrophic failures. Cooperation thus becomes the optimal strategy and absolutely outperforms defection in the game evolution with respect to the "death" mechanism. Our results can be useful for understanding large-scale catastrophe in real-world systems and in particular, they may yield insights into significant social and economical phenomena such as large-scale failures of financial institutions and corporations during an economic recession. © *2011 American Institute of Physics*. [doi:10.1063/1.3621719]

Evolutionary games are a powerful paradigm to study a variety of self-organized behaviors in natural, social, and economical systems. So far, the tolerance of individuals to elimination or death in the paradigm has received relatively little attention. A relevant example is the bankruptcy of agents in an economical system. For any agent, a lowest amount of profit should be maintained for it to survive, which comes from the interactions with other agents in a certain time period for continuous investment into the future. Another example is ecosystems, where individuals compete and cooperate for essential life-sustaining resources. If some minimal requirement for resources cannot be satisfied, individuals will die. In this paper, we incorporate an elimination mechanism into the gaming rules to better mimic the evolution of cooperative behavior in realistic systems. In particular, we assign a tolerance parameter to every individual in the network, which is the lowest payoff needed for an individual to survive. Taking into account the diversity in social and biological systems, we assume that each individual can have its own tolerance. For example, the number of interactions of an individual is a characteristic to distinguish it from others, so it can be used to define the tolerance. In a network, the death of an individual leads to the removal of its nodes together with all its connections with the others. The game and network thus co-evolve as a result of elimination according to the survival tolerance. Our main finding is that, in the presence of defectors, a cascading process of death of individuals can occur in relatively short time, which can even spread to the whole network, leading to complete extinction. Strikingly, we find that a pure cooperation state can emerge after the cascade terminates, in which the exclusive survivors are cooperators. This phenomenon occurs regardless of the type of games and of the network topology. This finding strongly suggests that defectors, despite their temporary advantages, are vulnerable to catastrophic cascading process. For an individual, cooperation becomes the optimal strategy to maximize benefit and avoid death. This resolves the social dilemma of profit versus cooperation in a natural manner. Our results can yield insights into the mechanism of catastrophic events in economical and ecosystems. For example, during the recent economic recession, a large-scale bankruptcy of financial organizations is a typical cascading process. For evolutionary biology, our result may provide hints to the mechanism of large-scale species extinction in a relatively short time period.

I. INTRODUCTION

A hallmark of the recent economical recession is the collapse and bankruptcy of a large number of financial institutions and corporations on a scale that has not been seen since the great depression. The manner by which the failures occur may be described as a cascading process, where the initial collapse of one or a few institutions, for example, triggered the failures of many others. While sophisticated economical and social models can conceivably be constructed to describe the process of cascading collapses, from the standpoint of physics, we are interested in a "minimal" model that can capture the major generic ingredients of the process, which are independent of the system details. It is hoped that the model can then lead to insights into the prevention of such cascades. The purposes of this paper are to construct such a model, to present analyses and numerical results, and to explore the implications.

For convenience, we shall refer to financial institutions, banks, and corporations etc., collectively as *agents*. In general, agents are connected with each other through a networked structure that often can be complex. While complex networks are common in natural and man-made systems, the interactions among the agents can also be quite complicated. In this regard, evolutionary games are appropriate and effective to describe the interactions among agents. The basic feature of our model is thus evolutionary games on large networks. In the following, we shall give a brief description of background on evolutionary games and existing works, justify a new feature that we introduce to model failures or elimination of agents, and state our findings.

Evolutionary game theory has been a powerful tool to study a variety of self-organized behaviors in natural, social, and economical systems.[1–4] A ubiquitous behavior is cooperation, which is necessary for generating and maintaining orders in these systems. Understanding how cooperation emerges among selfish individuals has been a challenging problem, especially in view of the dilemma in social science that disfavors cooperation.[5–8] Prisoner's dilemma games (PDGs),[9] snowdrift games (SGs),[10] and public goods games (PGGs)[11] have been used to model interactions among selfish individuals and how the social dilemma can be resolved through self-organization. In particular, PDG and SG are two-player games, while PGGs are usually played by groups of agents. SG, also known as the Hawk-Dove game, is more favorable for the emergence of cooperation than PDG. Based on these games, a number of mechanisms have been discovered that facilitate cooperation, which include reputation and punishment,[12–17] network reciprocity,[18–20] success-driven migration,[21–24] memory effect,[25,26] noise,[27,28] teaching ability,[29–33] social diversity,[34–36] asymmetric cost,[37] etc. In most of these works, since the focus was on the emergence of cooperation, it is not necessary to incorporate any failure mechanism. That is, during the process no individual agent is eliminated from the game. Apparently, this assumption is not suitable for situations where agents can go bankrupt.

In existing works on evolutionary games on networks,[18–20,34,38,39] agents' payoffs are determined by both their and their opponents' strategies according to a certain set of rules. All agents imitate their neighbors to acquire strategies to gain more payoffs, where the neighbors of an agent are those that are directly connected to it, as determined by the network topology. We shall incorporate a failure or an elimination mechanism into the gaming rules. In particular, we assign a tolerance parameter to every agent in the network, which is the lowest payoff needed for the agent to survive. Due to diversity in the properties of the agents, this parameter can be different for different agents. To define the tolerance, we follow the phenomenology of the crisis of financial markets in the great recession. For any profitable agent, a lowest amount of profit should be maintained, which comes from the interactions with other agents in a certain time period for continuous investment into the future. The minimal value of the profit is related to the size of the agent: larger organizations require higher profits to maintain their normal functioning and to avoid bankruptcy. An appropriate definition of the tolerance with respect to the diversity of agents can be obtained by using the normal payoff of agents as a reference, which is the payoff gained in a healthy market where all agents are cooperators. Since information about the neighborhood has been incorporated into the normal payoff, information about the diversity of agents is naturally embedded in the tolerance. When the lowest payoff of an agent cannot be achieved during the game, agent becomes bankrupt and is removed, together with all its links, from the network.

So far, death and removal of agents in evolutionary games have received little attention and there has been no work on sudden, cascading-like large-scale failures of agents. For example, in some previous studies, the death of an agent is usually accompanied by the birth of a new one at the empty site, resulting from the competition in the neighborhood[15,40,41] or from imitating strategies of those neighbors with higher fitness. While the assumption of simultaneous death and birth is suitable for addressing issues such as whether natural selection favors cooperation and what topology promotes cooperation, it is not suitable for studying cascading failures. Our working hypothesis is then that death of agents is much faster than the generation of new ones in a certain time period, so that the birth process can be effectively neglected. That is, when an agent is removed from the game, the site in the network that it originally occupies remains empty through the dynamical process. This is a key difference between our model and previous ones, which allows us to investigate catastrophic behavior through a "minimal" model. Our model also differs much from previous ones of games on adaptive networks[42–44] and of the evolution of cooperation under topological attacks or errors.[45]

Our main finding is that in the presence of defectors so that agents are vulnerable to defection for instantaneous higher payoff, a cascading process of agent death can occur in relatively short time, which can even spread to the whole network, leading to complete extinction. This result captures the essential feature of what happened in financial markets during the recent economic recession, where the defection strategy can generally be represented by several types of harmful economical activities, such as sub-prime mortgage securities. Such high-risk investments decrease the capacity of agents to resist defection. Our finding implies that deleterious economical strategy can play a key role in the outbreak of bankruptcy cascades, which is consistent with intuitive understanding. Strikingly, we find that a complete cooperation state emerges after the cascade terminates and the exclusive survivors are cooperators, which holds regardless of the type of games or of the network topology. This finding strongly suggests that defectors, although they can gain much more payoffs and prevail temporally, are extremely vulnerable to the occurrence of the catastrophic cascading behavior. For an agent, in order to maximize its payoff while avoiding death, cooperation becomes the optimal strategy. When most agents cooperate, the system can be maintained in a healthy state in that no large-scale cascading events are likely. Our finding suggests that rational agents can certainly survive and make profit through cooperation, which naturally resolves the social dilemma of profit versus cooperation.

A brief account of some of the results with a focus on regular networks has appeared recently.[46] The contributions of this paper are (1) to provide extensive numerical results and a detailed theoretical analysis of cascading failures in regular networks, (2) to extend the computation and theory to complex networks with details, and (3) to investigate cascading dynamics triggered by a single defector. Compared with our recent brief work,[46] in this paper, issue (1) is investigated at a significantly more detailed level, issue (2) is almost new, and issue (3) is entirely new.

This paper is organized as follows. In Sec. II, we detail our model. Numerical results and analyses for regular and complex networks are presented in Secs. III and IV, respectively. In Sec. V, we treat the case where cascading failures are triggered by a single defector. Conclusions and discussions are offered in Sec. VI.

II. MODEL

We shall use all three types of games (PDG, SG, and PGG) studied commonly in the literature to simulate interactions among agents in the network. Our goal is to identify generic dynamical features that hold regardless of the type of the game. The main ingredients of these games are as follows. (1) In PDG, there are two players and they can choose either to cooperate or to defect. Both players are offered a reward R for mutual cooperation and a lower payoff and a punishment P for mutual defection. If one player decides to cooperate but the other defects, the defector gets the highest payoff T (temptation to defect), while the cooperator gets the lowest payoff S. The payoff rank for PDG is, thus, $T > R > P > S$. (2) In SG, there are also two players but the payoff rank is $T > R > S > P$, where the positions of S and P are reversed as compared with PDG. This means that mutual defection is an irrational strategy in SG. (3) PGG differs from PDG and SG in that it is played by a group of players. In such a game, cooperators contribute a cost c to the public good and defectors do nothing. The total reward is the product between the total contribution and an enhancement factor η, which is equally distributed among all members in the group. Thus, a defector's payoff without cost is always larger than a cooperator's.

The particular sets of players involved in a game are determined by the network topology. At each time step, the actual payoff gained by any agent is the sum of payoffs resulting from all interactions with others. Initially, each node of a connected network is occupied by either a cooperator or a defector. At each iteration (time step), there are three dynamical processes.

(1) *Game playing and payoffs*. For PDG, we follow previous studies and use the rescaled parameters $R = 1$, $T = b$ $(b > 1)$, and $S = P = 0$.[18] Thus, b is the only parameter. Reference 18 also shows that the condition $P = \varepsilon$, where ε is positive but significant below unity, gives exactly PDG dynamics. For SG, we set $R = 1$, $T = 1 + r$, $S = 1 - r$, and $P = 0$, so the single parameter is $0 < r < 1$.[20] For PGG, in an arbitrary group formed by node x and its neighbors, the payoffs of a defector and a cooperator are

$$P(D) = \frac{c\eta n(C)}{k_x + 1} \quad \text{and} \quad P(C) = P(D) - c, \qquad (1)$$

respectively, where η is the enhancement parameter, $n(C)$ is the number of cooperators in the group, and k_x is the number of neighbors of node x. Without loss of generality, we set c to be unity.[35] At each step, an arbitrary individual x is involved in $k_x + 1$ PGGs centered at x and its neighbors.

(2) *Failure and agent removal*. At each iteration, the node that hosts agent i and all its links will be removed, if

$$P_i < T_i \equiv \alpha P_i^N, \qquad (2)$$

where P_i is the actual payoff of agent i gained from all interactions with others, T_i is the tolerance to death, P_i^N is the normal payoff when the system is in a healthy state in which all agents are cooperators, and $0 \le \alpha \le 1$ is a tolerance parameter. For PDG and SG, we have

$$T_i = \alpha P_i^N = \alpha k_i, \qquad (3)$$

where k_i is the number of neighbors of i at the beginning. For PGG, we have

$$T_i = \alpha P_i^N = \alpha(\eta - 1)(k_i + 1). \qquad (4)$$

For $\alpha = 1$, agents have zero tolerance to breakdown, while for $\alpha = 0$, agents are completely tolerant.

(3) *Strategy updating*. At each time step, agent i randomly chooses a survived neighbor j and imitates j's strategy with the probability[19,20]

$$W_{i \to j} = \frac{1}{1 + \exp[-(P_j - P_i)/\mathcal{K}]}, \qquad (5)$$

where $\mathcal{K}$ is the level of "noise" representing the uncertainties in assessing the payoffs. In our simulations, we set $\mathcal{K} = 0.1$ (quite arbitrarily).

III. CASCADING FAILURES AND SURVIVAL STRATEGY ON REGULAR NETWORKS

We consider lattices with varying numbers of neighbors for each site under periodic boundary conditions. Figure 1(a) shows, for a two-dimensional lattice where each site has four neighbors (2D4n), a typical time series of the number n_d of failed (or dead) agents, defined as the number of removed nodes normalized by the network size. Time evolutions of the fractions of cooperators ρ_C and of defectors ρ_D among survivors are shown in Fig. 1(b). We see that a cascading process of failures occurs, where about 80% of the agents eventually fail and are removed. Associated with the death of agents, ρ_c first decreases with time and then reaches unity after the cascading process is complete and the system reaches a new steady state. In contrast, after a small increment at the beginning of the cascading process, ρ_D decreases continuously and tends to zero eventually. These results indicate that, after a cascading process, cooperators are the sole survivors. Similar results have been obtained for SG and PGG. The general observation is that, when defecting

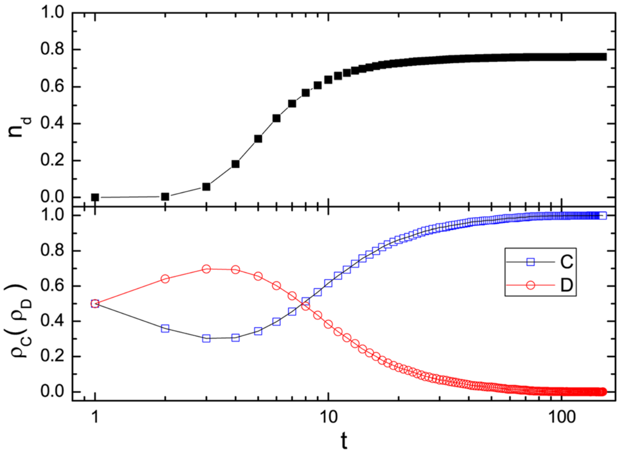

FIG. 1. (Color online) For PDG on a two-dimensional lattice with four neighbors for each site, time series of the number n_d of dead agents and the fractions of cooperators ρ_c and defectors ρ_D in the survivors for $\alpha = 0.5$ and $b = 1.1$. Initially, same numbers of cooperators and defectors are randomly distributed over the entire lattice.

strategies are practiced, the occurrence of large-scale cascading processes is common and, in order to survive, an agent needs to cooperate persistently. Our computations have also revealed that the value of n_d depends on the tolerance parameter α, the temptation to defect (b or r), and the enhancement factor η. If defections occur significantly more often than cooperations, a complete breakdown of the system is likely, where no agent can survive.

To gain insights into the cascading process, we examine the evolution of spatial patterns. Figures 2(a)–2(e) show, for PDG, spatial patterns at five instants of time. Initially 10% of agents are chosen to be defecting and randomly placed in the ocean of cooperators [Fig. 2(a)]. At some early stage, there is an increase in the number of defectors but empty sites begin to arise around defectors, and both cooperators and defectors begin to die, as shown in Fig. 2(b). At a later time, the death rate of defectors exceeds that of cooperators and clusters of cooperators begin to form, as shown in Fig. 2(c). Some time later, only a small number of defectors at the boundary of cooperator clusters are still alive, as shown in 2(d). Finally, defectors become extinct and the lattice is shared by cooperator clusters and empty sites [Fig. 2(e)]. After the extinction of defectors, the death of cooperators stops and the pattern becomes time-invariant.

The evolution of spatial patterns leads to a qualitative explanation for the outbreak of death and survival of cooperators. In particular, the spread of the defection strategy and the loss of interactions among agents induce continuous death and emergence of empty sites adjacent to defectors, but the increase of empty sites and the formation of cooperator clusters lead ultimately to extinction of defectors. This can be explained by focusing on the interaction among defectors and their neighbors. Suppose that, a defector is surrounded by cooperators. The defector's payoff will be the highest and larger than the tolerance value. However, two situations may arise that can cause the defector to die. First, due to the highest payoff gained by the defector, cooperating neighbors tend to imitate its strategy and betray with a high probability. Second, death of cooperating neighbors can occur because of their insufficient cooperations. Regardless of which situation actually occurs, a negative feedback mechanism is induced by the defector, resulting in the reduction of its payoff. Consequently, either the defector becomes vulnerable to death or it is overwhelmed by neighboring cooperators. For cooperating neighbors, once they turn to be defectors, the defection strategy spreads and further death can follow until the emergence of cooperator clusters. At the boundary of the clusters, cooperators receive sufficient mutual cooperations to resist both invasion of defectors and insufficient payoffs to death. Defectors adjacent to the boundary are surrounded by many empty sites and cannot gain enough payoffs from cooperators to survive.

Figures 3(a)–3(c) show representative final patterns for two-dimensional lattices with 4 and 8 neighbors. We denote the two network systems by 2D4n and 2D8n, respectively, and will use a similar notation for other cases treated in this paper. When the initial fraction of defectors is small and the tolerance to death is high, a large number of cooperators can survive and they tend to form large areas of clusters, as shown in Fig. 3(a) for a 2D8n lattice. For high temptation to defection and low tolerance, only small groups of cooperators can survive in the sea of vacant sites, as shown in Fig. 3(b) for a 2D8n lattice and Fig. 3(c) for a 2D4n lattice. We see that, even when failures are massive, there can still be small clusters of cooperators that survive the catastrophe. As

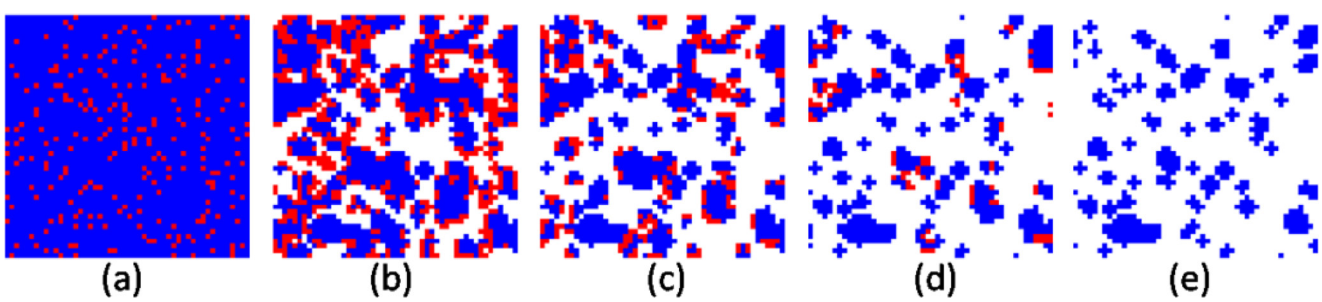

FIG. 2. (Color online) For PDG on 2D4n, evolution of spatial patterns for $b = 1.2$. The lattice size is 50×50 and all sites are occupied initially. (a) For $t = 0$, 10% of the occupants are defectors randomly distributed on the lattice. The color coding is red (light gray) for defectors, blue (dark gray) for cooperators, and white for empty sites. (b) For $t = 12$, defectors reproduce themselves and invade the domain of cooperators. Note that empty sites arise near the defectors. (c) For $t = 21$, the number of defectors has been reduced considerably, the vacant areas enlarge, and some clusters of cooperators begin to form. (d) For $t = 33$, only a small number of defectors at the boundary of cooperator clusters are still alive. (e) For $t = 86$, all defectors have failed and have been removed, marking the end of the cascading process. For $t > 86$, the spatial pattern is invariant and the number of survivors is a constant.

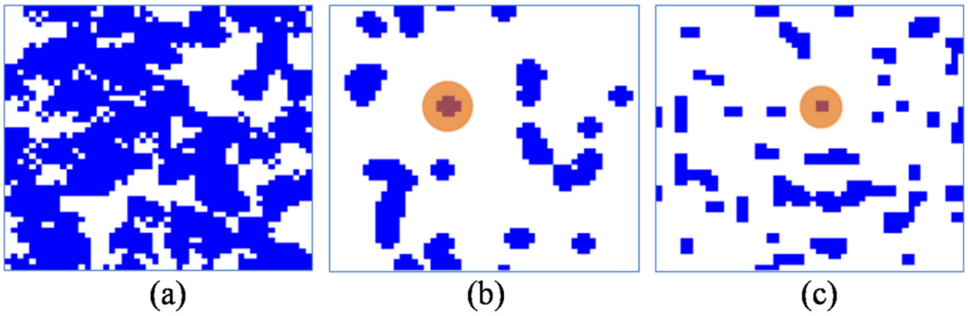

FIG. 3. (Color online) Representative spatial patterns for PDG on two-dimensional lattices with 8 and 4 neighbors, where the initial state is that 10% of the agents are defectors: (a) for a lattice with 8 neighbors (2D8n), a small cascade of death with a large fraction of cooperators surviving finally for $b = 1.01$ and $\alpha = 0.1$; (b) for the same lattice but for $b = 1.05$ and $\alpha = 0.4$, a number of small clusters of cooperators that remain even when death of agents is severe; and (c) for a 2D4n lattice for $b = 1.05$ and $\alpha = 0.4$, small surviving clusters of cooperators. In (b) and (c), two typical cooperator clusters are marked, which are the smallest clusters surviving through the cascading process.

we will discuss below, for a regular lattice, a sudden transition from survival state to extinction can occur, and the transition point is determined by nothing but the stabilities of these small clusters.

To obtain a better understanding of the cascading dynamics, we investigate the dependence of the size (fraction) of dead agents, s_d, on the tolerance parameter α for three games on four types of lattices with two, four, six, and eight neighbors for each site (denoted by 1D2n, 2D4n, 2D6n, and 2D8n, respectively). The quantity s_d is defined to be the number of removed agents normalized by the network size N after cascading failures cease. The basic cells of these lattices are shown schematically at the top of Fig. 4, and the computational results in Fig. 4 are with respect to PDG. We observe the appearance of step structures for all lattices, where, for such a step, s_d changes discontinuously from one constant value to another. For different lattices, the numbers of steps are different. Since all survivors are cooperators, N_c (the number of cooperators) as a function of α displays step structures as well because of the simple relation $N_c = N(1 - s_d)$. A striking phenomenon is that the transition from a survival to an extinction state occurs at the critical value $\alpha_c = 0.5$, regardless of the lattice type and of other parameters such as the temptation to defection and the initial fraction of defectors. Similar results have been found for SG with the same value of α_c. For PGG, because of the intrinsic group interactions, the behavior of s_d versus α is somewhat different from those with PDG and SG. However, the phenomenon of transition to extinction persists, as shown in Fig. 5. We observe that, except for 1D2n lattice, there are no clear step structures and the transition points differ for different lattices.

To explain the transition to extinction, we focus on the stabilities of various surviving clusters of cooperators. For instance, we can study such clusters for parameter α slightly below the critical value 0.5. The structures of the "minimal" clusters, one for each lattice type, are shown schematically in Fig. 6. Their stabilities can be assessed by calculating the payoffs of agents in the respective clusters. For example, for 1D2n lattice, the two cooperators' payoff is $P_i = 1$ and their

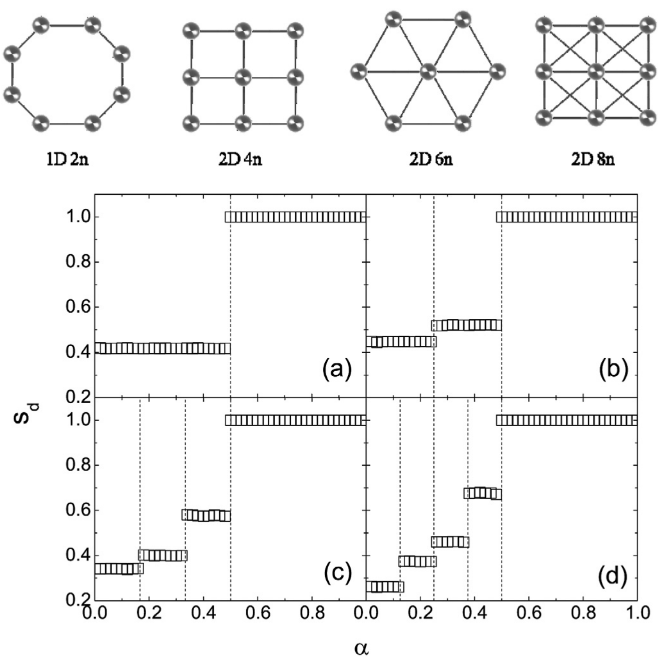

FIG. 4. For the PDG on four types of regular networks: (a) 1D2n, (b) 2D4n, (c) 2D6n, and (d) 2D8n, as schematically illustrated at the top, the fraction of failed (dead) agents, s_d, as a function of the tolerance parameter α. The dashed vertical lines are theoretical predictions for various transitions between distinct states, including the extinction transition. The network size is 100×100 and all data points are obtained after a steady-state is reached for which s_d remains to be a constant.

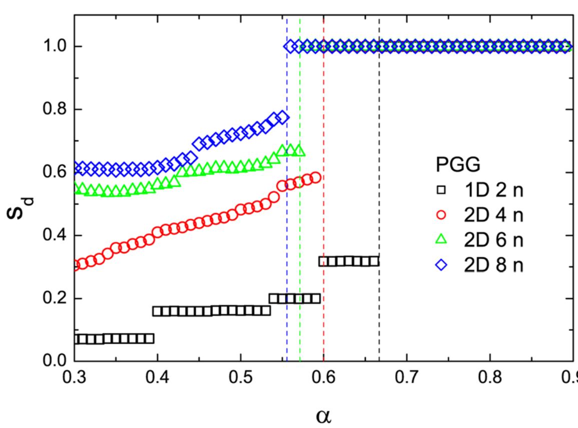

FIG. 5. (Color online) For the PGG, dependence of s_d on the tolerance parameter α for the four types of networks as in Fig. 4. The dashed vertical lines are theoretical predictions for the extinction-transition points for the four types of networks, as given by Eq. (8). The network size is 100×100.

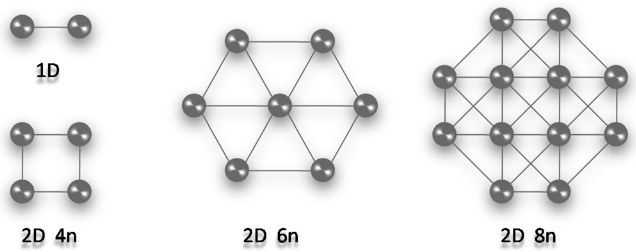

FIG. 6. Four smallest surviving clusters in four types of lattices for α slightly below the critical value α_c for all three games. These clusters determine the transition point to extinction. The stabilities of nodes inside the clusters can be determined by comparing their remaining payoffs with their tolerance payoffs. The nodes inside the cluster are more stable than those at boundaries. For $\alpha > \alpha_c$, nodes at boundaries die out and the clusters disappear.

tolerance is $T_i = \alpha$, $k_i = 2\alpha$. For $\alpha < 0.5$, we have $P_i > T_i$, so that both cooperators will survive and the cluster is stable. Similarly, all clusters in Fig. 6 are stable for $\alpha < 0.5$. For $\alpha > 0.5$, these clusters become unstable and there are no longer survivable structures.

The transition in PGG can be understood similarly. For an arbitrary surviving node i, its payoff satisfies $P_i > T_i$. Combining Eqs. (2) and (4), we have

$$P_i = (\eta - 1)(k_i' + 1) > \alpha(\eta - 1)(k_i + 1) = T_i, \qquad (6)$$

where k_i is the original degree of i and k_i' is the remaining degree in the aftermath of the cascading event. The critical value α_c for the transition point is then given by

$$\alpha_c = \frac{k_i' + 1}{k_i + 1}, \qquad (7)$$

which is independent of the enhancement parameter η. Since for PGG, the smallest stable cluster has the same structure as that for PDG, the transition point α_c is also determined by the cluster structure in Fig. 6. A common property among these minimal cluster structures is that the remaining degree k_i' for any node is large than or equal to $k_L/2$, where k_L is the degree of the node in the original lattice. Since nodes with more remaining connections are more stable for identical original degrees, the extinction-transition point α_c is determined by the nodes at boundary. We then have

$$\alpha_c = \frac{k_L + 2}{2(k_L + 1)}. \qquad (8)$$

The values of α_c for the four types of lattices can then be calculated as $\alpha_c^{1D2n} = 2/3$, $\alpha_c^{2D4n} = 3/5$ $\alpha_c^{2D6n} = 4/7$ and $\alpha_c^{2D8n} = 5/9$. These predictions are verified by simulation results for PGG, as shown in Fig. 5.

We can consequently explain the presence of step structures in Fig. 4 associated with the transitions from one surviving state to another by examining the condition for survival,

$$P_i = k_i' > \alpha k_i = T_i, \qquad (9)$$

or

$$\alpha_c = \frac{k_i'}{k_i}. \qquad (10)$$

Because the remaining degree k_i' satisfies $k_i' \le k_i$, its possible values are $1, 2, \ldots, k_i$. However, since no stable cluster exists for $\alpha > 0.5$, there is an additional constraint for k_i': $k_i' \le k_i/2$. All possible values of k_i' determine the numbers of steps in Fig. 4. For 1D2n lattice, $k_i' = 1$ is the only choice so that $\alpha_c = 1/2$, which separates two steps in s_d. For 2D4n lattice, k_i' can be 1, 2, which results in $\alpha_c = 1/4$ and $1/2$, corresponding to 3 steps. Similarly, for the 2D6n lattice, we have $\alpha_c = 1/6$, $1/3$, and $1/2$, which separate the whole α-interval into 4 steps. For 2D8n lattice, we have $\alpha_c = 1/8$, $1/4$, $3/8$, and $1/2$, so there are 5 steps. These predictions are in good agreement with numerical computations, as shown in Fig. 4.

IV. CASCADING FAILURES ON COMPLEX NETWORKS

A. Scale-free networks

Extensive research in the past decade has revealed that a large number of real-world networks possess the small-world and/or the scale-free topology.[47–49] The small-world topology is especially relevant to social and economical networks where node-to-node interactions are best described by evolutionary games. It is thus important to test whether our finding from regular networks in Sec. III, namely, that defection strategy can lead to large-scale cascading failures and cooperators are the sole survivors, applies to complex networks.

For evolutionary games on complex networks, it has been established that cooperation can be supported by several natural mechanisms, for example, repeated interactions,[5] punishment,[12–17] and migration.[21] It turns out, counter-intuitively, that heterogeneity in node degrees can be beneficial to the emergence and persistence of cooperation both for two-player games (PDG and SG)[34] and for games involving groups of players (PGG).[35] To make an unbiased comparison with our results from regular networks, we shall implement our evolutionary-game model in Sec. II on both scale-free and small-world networks. For scale-free networks, we use the standard Barabási-Albert model,[50] while for small-world networks, we use the Newman-Watts model.[51] We focus on how the size of death and the surviving strategies depend on two key game parameters, α and $b(r)$ or η, as defined in Sec. II.

Figures 7(a)–7(c) show the contour plots of s_d in the two-dimensional parameter plane for PDG, SG, and PGG on scale-free networks, respectively. Analogous to what has been observed on regular lattices, there exist two exclusive asymptotic phases: extinction without any survivors for large values of α and $b(r, \eta)$ and a survival phase in which only cooperators can resist death and survive, regardless of the values of α and $b(r, \eta)$. Cooperators and defectors cannot coexist for any parameter combinations. Similar to what we have observed for regular lattices, the defectors are destined to be eliminated despite their temporal high advantage when encountering cooperators. Thus, the finding that defection can only bring short-term benefit to agents with respect to cascading failures, and cooperation is the essential strategy for survival, holds true also for complex networks. A difference from the case of regular networks is that, for complex networks, s_d is more sensitive to the variation of α and the s_d -versus-α curve tends to be continuous. This is understandable considering that the underlying complex topology

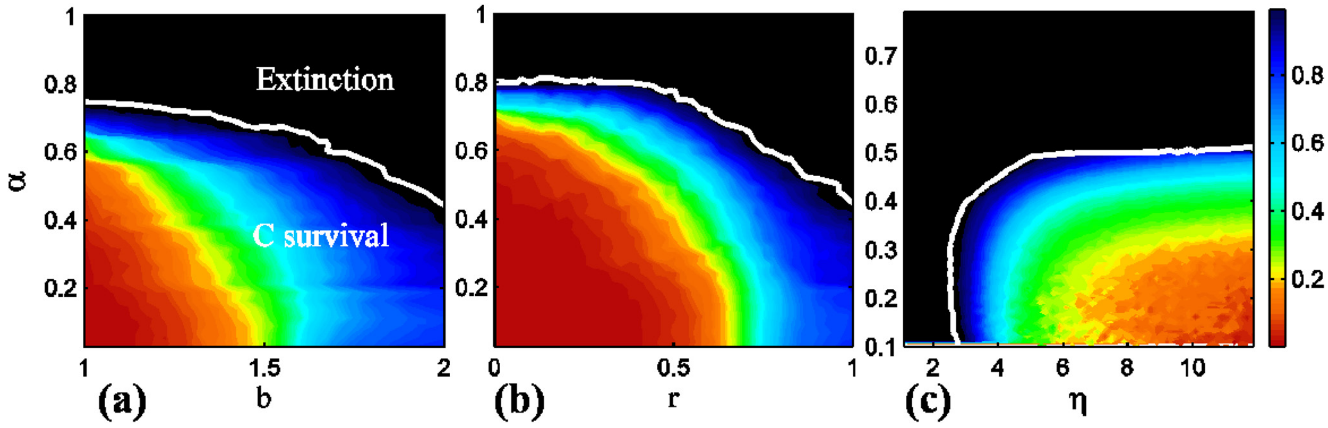

FIG. 7. (Color online) For the PDG, SG, and PGG on scale-free networks, dependence of death size s_d on the tolerance parameter α and game parameters b, r, and η, respectively. The initial fraction of defectors is 0.1 for PDG and SG and 0.5 for PGG. There are two distinct asymptotic phases: extinction and survival of cooperators. The boundaries between the two phases are marked by the white curves. In the entire parameter space, defectors cannot survive. For the three games, larger values of α lead to larger values of s_d because agents are more vulnerable to payoff decrease from losing connections with or being betrayed by neighbors. Failures, however, can be made less severe by reducing the temptation to defection. Decreasing $b(r)$ or increasing η can enhance the formation of cooperation clusters and their abilities to resist both death and invasion of defectors. Ensemble average is based on 10 network realizations and 10 independent gaming processes for each network realization. The size N of the scale-free network is 1000 and the average degree $\langle k\rangle$ is 6. The degree distribution is $P(k)\sim k^{-3}$ for $N\rightarrow\infty$.

provides a richer spectrum of agent tolerances due to diversity in the node degrees. This difference appears mostly at a detailed level and it does not affect our general conclusion on cascading failures and surviving strategy, which holds regardless of the game type and of the network topology.

The boundary between extinction and cooperator survival for scale-free networks can be estimated, as follows. Because of the highly heterogeneous degree distribution and the associated complex connecting pattern, stable clusters resistant to extinction can no longer be expected. The boundary in fact depends on parameters (e.g., b and r for PDG and SG, and η for PGG). However, in certain regimes, the parameter dependence can be weak. For example, we find that for PGG, the boundary hardly depends on η if it is large. In this case, the boundary is solely determined by the network structure, which can then be treated by a stability analysis as we have done for regular networks. This is particularly the case when cooperation is facilitated by large values of η.

To proceed, we note that, for a scale-free network, vast majority of the nodes have in fact very small degrees. The stabilities of these nodes can play a key role in the extinction. In reality, the number of connections of an agent determines its robustness. We can thus assume that all agents with the smallest degree (k_{min}) have the identical critical tolerance to death, α_c, where for $\alpha>\alpha_c$, the death of these agents can possibly trigger an extinction event due to their large numbers. The critical tolerance of agents with k_{min} can thus be used to estimate the extinction boundary. This approach also makes use of the fact that hubs are more stable than small-degree agents in that the latter usually fail more easily than the former. From Eq. (7), the stability condition for the smallest-degree agents in PGG can be written as

$$\alpha_c=\frac{k'_{min}+1}{k_{min}+1},\qquad(11)$$

where k'_{min} is the minimal degree of the network after cascading. The range of possible values of k'_{min} is from 1 to k_{min}, which depends on both the enhancement parameter η and the initial fraction of defectors. However, when these two parameters assume large values, only the most stable agents can survive. An agent can survive when there is at least a single interaction. We thus have $k'_{min}=1$. If α is reduced such that the single interaction cannot provide enough payoff for the agent to sustain, extinction will arise. For the standard scale-free network,[50] the average degree is $\langle k\rangle=2k_{min}$. The extinction threshold thus depends on $\langle k\rangle$ and can be written as

$$\alpha_c=\frac{4}{2+\langle k\rangle}.\qquad(12)$$

Similarly, for PDG and SG, we have

$$\alpha_c=\frac{2}{\langle k\rangle},\qquad(13)$$

which is valid in the regime of small temptation to defection and large initial fraction of defectors. Figure 8 shows these theoretical estimates together with results from direct numerical simulations, where a good agreement is observed. Our computations also reveal that, for scale-free networks, the stable cluster possesses a star-like structure, as indicated in Fig. 8, where all connections are originated from a hub node. For $\alpha>\alpha_c$, the star-like structure becomes unstable and no agents can survive, signifying onset of extinction.

The cascading size of death, s_d, is not sensitive to noise $\mathcal{K}$, as shown in Fig. 9. This is because of the fact that the time scale of the cascading process is faster than the scale of strategy updating process, so that noise in the probability of strategy updating has little influence on s_d.

B. Small-world networks

The contour plots of extinction and survival regions in the parameter space for PDG, SG, and PGG on Newman-Watts small-world (NW) networks are shown in Fig. 10,

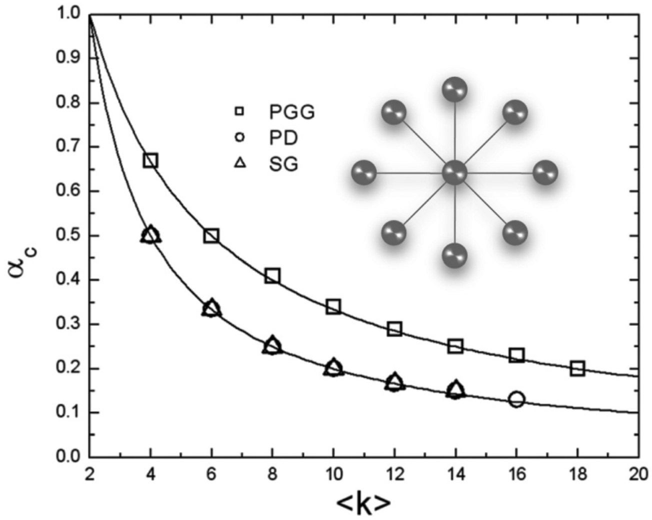

FIG. 8. For the standard scale-free network, extinction boundary α_c as a function of the average degree $\langle k \rangle$ for the three types of games, where the initial fraction of defectors is 0.85 (relatively high) and the temptation-to-defection parameter is $b = 1.01$ and $r = 1.01$ (for PDG and SG, respectively) or the enhancement parameter is $\eta = 10$ (for PGG). The network size is 1000. Each data point is obtained by averaging over 10 network realizations and 10 independent gaming processes for each network realization. The star graph is a typical survivable cooperator cluster for α close to the boundary α_c.

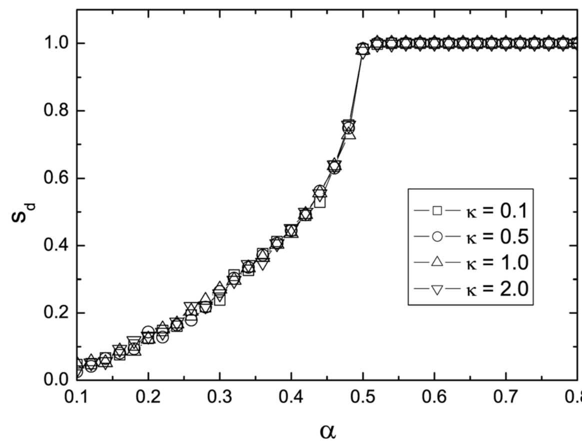

FIG. 9. For PGG, dependence of s_d on tolerance parameter α for different values of noise level $\mathcal{K}$ on scale-free networks. The average degree $\langle k \rangle$ is 10, network size is 1000, and the enhancement parameter η is 10.

respectively. Similar to the observations from lattices and scale-free networks, there are two phases: extinction and survival, which are separated by the white curves. In the survival phase, cooperators are the only survivors. The survival region is smaller compared to that on scale-free networks, due to the fact that the scale-free structure tends to promote cooperation.

V. CASCADING FAILURES TRIGGERED BY A SINGLE DEFECTOR

A. Regular lattices

Here, we discuss the scenario in which a cascading process of death is triggered by a single defector. In Fig. 11, we show some typical spatial patterns on regular lattices starting from a central defector using PDG as an example. Figures 11(a)–11(c) are for a 2D4n lattice. In Fig. 11(a), for small b and large α, e.g., $b = 1.1$ and $\alpha = 0.8$, the single defector results in continuous death starting from the central site and all agents become extinct eventually. In this case, the death process is induced exclusively by the loss of interactions from the removal of neighbors (not by the diffusion of defection strategy). In Fig. 11(b), for large b and small α, during the cascading process, most agents die except a few small clusters that are stable enough to survive. In contrast, for small b and small α values, as shown in Fig. 11(c), more cooperator clusters of larger sizes survive as compared to (b). In Figs. 11(b) and 11(c), large-scale death is triggered by both diffusion of defection strategy and loss of interaction with cooperators among agents. Figure 11(d) exhibits the spatial pattern on a 2D8n lattice for $b = 1.3$ and $\alpha = 0.4$. The survival clusters appear different from that for 2D4n lattice and the pattern of defectors invading the cooperator clusters is different as well.

The contour plot for PDG on 2D4n lattice with a single defector is shown in Fig. 12. An interesting phenomenon is that the cascading process of death is prohibited in the middle range of the threshold parameter α. This means that the robustness of the network system is a non-monotonic function of the agents' tolerances and, as a result, strong tolerance can lead to extinction more easily. To explain this counterintuitive behavior, we investigate the evolution of a sample lattice, as shown in Fig. 13. For simplicity, we

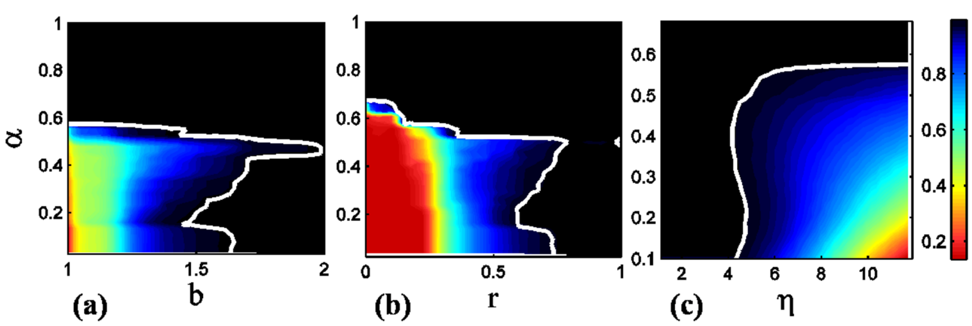

FIG. 10. (Color online) Dependence of the size of death, s_d, on the tolerance parameter α and the game parameter b, r, and η for PDG, SG, and PGG, respectively, on NW small-world networks. For PDG and SG, the initial fraction of defectors is 0.1. For PGG, it is 0.5. There are two exclusive phases: extinction and cooperator survival. In each case, the boundary between them is marked by the white curve. The network size is 1000. The coordinate number of initial ring is 3, and with probability 0.3, there is a link from each node connecting to a randomly picked node. The average degree $\langle k \rangle$ is 6.6.

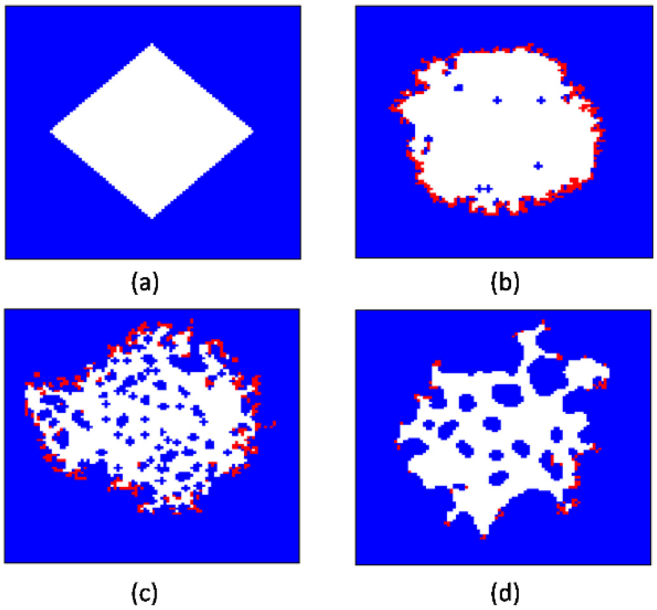

FIG. 11. (Color online) Representative spatial patterns for PDG with a single defector at the center on regular lattices: (a) $b = 1.1$, $\alpha = 0.8$, and $t = 35$; (b) $b = 1.8$, $\alpha = 0.25$, and $t = 93$; (c) $b = 1.1$, $\alpha = 0.1$, and $t = 128$; and (d) $b = 1.3$, $\alpha = 0.4$, and $t = 119$, where (a), (b), and (c) are for 2D4n and (d) is for 2D8n. The blue (dark gray) regions represent cooperators, the red (light gray) regions are for defectors, and the white regions denote empty sites.

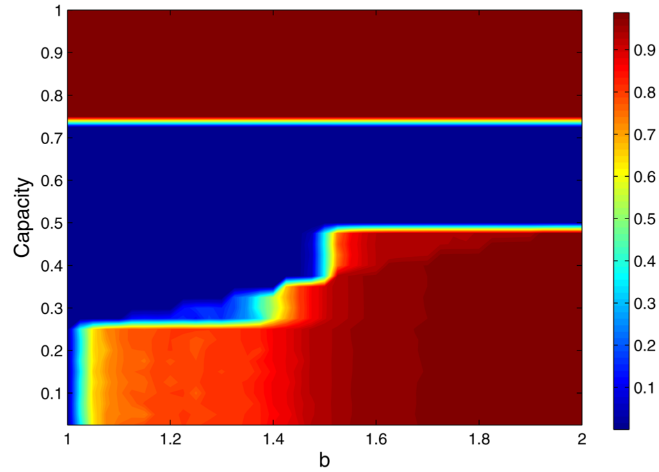

FIG. 12. (Color online) For a 2D4n lattice, dependence of the size of death s_d on the tolerance parameter α and game parameter b.

consider large values of b. In Fig. 12, for $0.5 < \alpha < 0.75$, nearly all agents survive, whereas extinction occurs in other regions of α. We thus study the role of α in three regions separately: (1) $\alpha < 0.5$, (2) $0.5 \leq \alpha \leq 0.75$, and (3) $\alpha > 0.75$. For $\alpha > 0.75$, according to the stability condition, a cooperator will die if it is in the vicinity of a defector or loses interaction with a cooperator neighbor. Thus, the cooperators adjacent to the defector die first. The defector cannot survive either and all agents will die eventually (Fig. 13). For $\alpha < 0.5$, at the first step, no agent dies. Instead, due to the advantages of defectors in gaining payoffs, the central defector will pass its strategy to its neighboring agents. After that the central defector cannot survive and the defection strategy spreads, together with the death of defectors except those at the boundary of the large defector cluster. Finally, all agents are eliminated. For the robust region $0.5 < \alpha < 0.75$, as shown in Fig. 13, we get from calculation that, at the first step, no agent dies and the central defector will pass its strategy to its neighbors. After that, since $\alpha > 0.5$, cooperators who have less than two cooperator neighbors will die. The surviving defectors' payoffs are less than those of their cooperator neighbors, preventing the spread of the defection strategy. At the next step, according to the survival condition, the defectors will die but their cooperator neighbors can survive. As a result, the death process ends, only 9 agents around the initial defector are eliminated, and the size of death as normalized by the total number of agents becomes negligible. We note that, for small values of b, for $\alpha < 0.5$, defection strategy

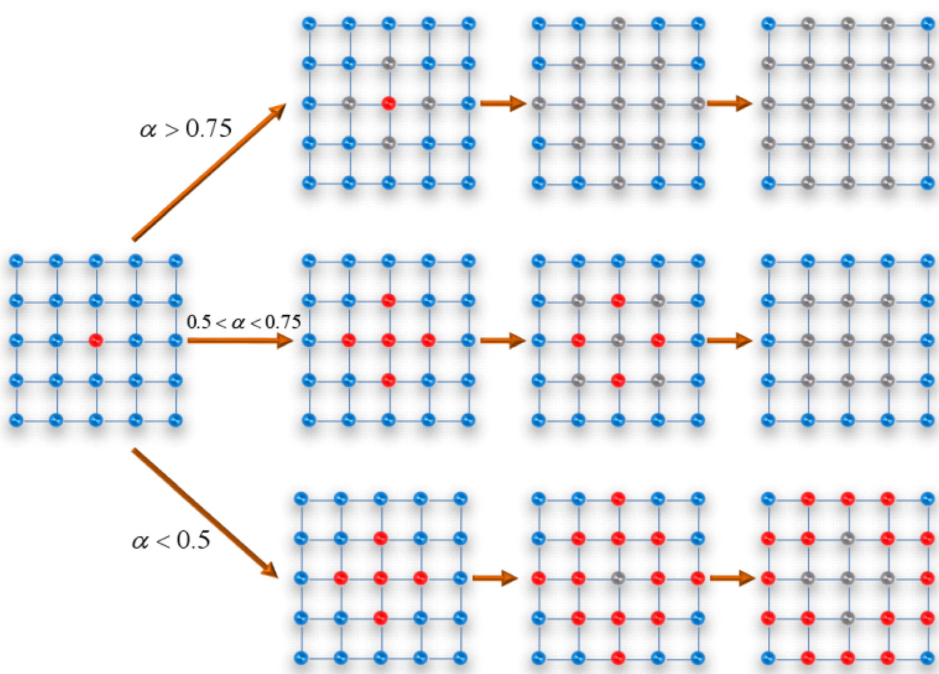

FIG. 13. (Color online) Illustration of the evolution from single defector in the ocean of cooperators on a 2D4n lattice. Blue (gray) represents cooperators, red (dark gray) signifies defectors, and light gray denotes dead agents. The examples are for large values of temptation to defect b. The counterintuitive results in Fig. 12 can be explained by the dynamical evolution on the sample lattice.

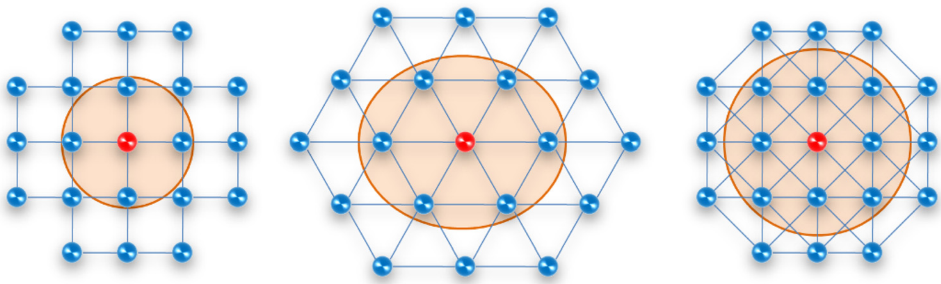

FIG. 14. (Color online) Three lattice configurations with single defector at center. The agents inside the circle determine the onset of the cascading process.

may not spread easily, leading to the shrink of the extinction region in Fig. 12 as compared to the case of large b.

An interesting issue then concerns the onset of cascading process triggered by a single defector on different regular lattices for PGG. We find two similar scenarios: (I) death of defector followed by the death of neighboring cooperators; (II) diffusion of defection strategy. These considerations lead to analytical insights into the onset of death for different types of lattices, which we now elaborate.

For 2D4n, the payoff of the defector as shown in Fig. 14 is

$$P_D = 5 \cdot \frac{4}{5}\eta, \tag{14}$$

where the payoff of the defector is collected from 5 groups. In each group, the defector's payoff is $4\eta/5$. The payoff of the cooperators in the vicinity of the defector is

$$P_C = 3(\eta - 1) + 2\left(\frac{4}{5}\eta - 1\right). \tag{15}$$

For case (I), if $P_C > P_D$, the defector cannot pass its strategy to neighboring cooperators, so only its death can trigger a cascading process. Using the condition $P_C > P_D$, we have

$$\eta > \frac{25}{3}. \tag{16}$$

In this case, the death condition for the defector is

$$T_D = 5\alpha(\eta - 1) > 4\eta = P_D, \tag{17}$$

which yields

$$\alpha > \frac{4\eta}{5(\eta - 1)}. \tag{18}$$

On the other hand, the death of the defector can lead to the death of neighboring cooperators to trigger a cascading process. The payoff of the neighboring cooperators after removing the defector is

$$p'_C = 4(\eta - 1), \tag{19}$$

and the death condition $T_C > P'_C$ yields $\alpha > 4/5$. Combining Eq. (19) with Eq. (18), we can get the critical value α_c for case I.

For case II ($\eta < 25/3$), diffusion of defection strategy can lead to death and removal of nodes. So the onset of the cascading process is determined by the survival probability of the cooperator after losing one connection. We then have

$$\alpha_c = \begin{cases} 4\eta/[5(\eta - 1)], & \text{if } \eta > 25/3; \\ 4/5, & \text{otherwise.} \end{cases} \tag{20}$$

For 2D6n lattice with a central defector, there are also two different situations, determined by $P_C > P_D$ and $P_C < P_D$, respectively, where

$$P_D = 6\eta, \text{ and } P_C = 3(\eta - 1) + 4\left(\frac{6\eta}{7} - 1\right). \tag{21}$$

The condition $P_C > P_D$ then leads to $\eta > 49/3$. Specifically for case I, we have

$$\begin{cases} T = 7\alpha(\eta - 1) > 6\eta = P_D, \\ T = 7\alpha(\eta - 1) > 6(\eta - 1) = P'_C, \end{cases} \tag{22}$$

which yields

$$\alpha > \frac{6\eta}{7(\eta - 1)}. \tag{23}$$

For case II, the critical value α_c can be obtained as

$$\alpha_c = \begin{cases} 6\eta/[7(\eta - 1)], & \text{if } \eta > 49/3, \\ 5/7, & \text{otherwise.} \end{cases} \tag{24}$$

For 2D8n lattice, we have

$$P_D = 8\eta \text{ and } P_C = 3(\eta - 1) + 6\left(\frac{8\eta}{9} - 1\right). \tag{25}$$

The inequality $P_C > P_D$ leads to $\eta > 21$. The critical value α_c for case (I) can then be obtained through

$$\begin{cases} T = 9\alpha(\eta - 1) > 8\eta = P_D, \\ T = 9\alpha(\eta - 1) > 8(\eta - 1) = P'_C, \end{cases} \tag{26}$$

which yields $\alpha > 8\eta/[9(\eta - 1)]$. Finally, we have

$$\alpha_c = \begin{cases} 8\eta/[9(\eta - 1)], & \text{if } \eta > 21, \\ 6/9, & \text{otherwise.} \end{cases} \tag{27}$$

Both the analytical predictions and numerical simulations are displayed in Fig. 15 for the three types of lattices. We observe an excellent agreement.

B. Scale-free networks with single defector on the largest hub

For scale-free networks, when the largest hub becomes a defector, the survival probabilities of a large number of

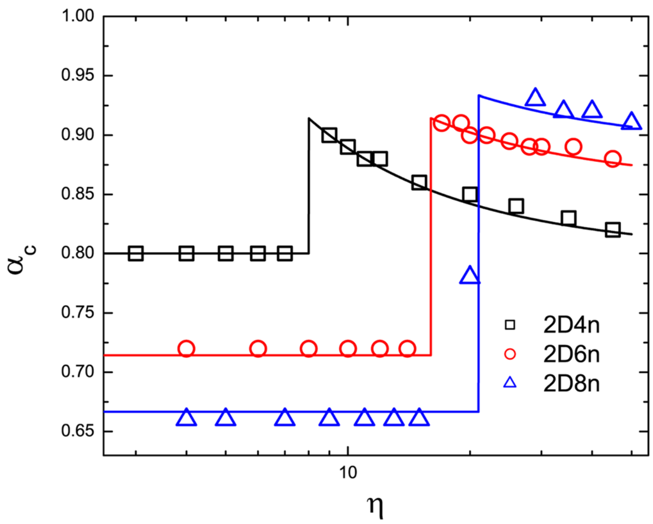

FIG. 15. (Color online) Critical value α_c of the onset of cascading process as a function of game parameter η for PGG for three types of lattices. The curves are theoretical predictions and data points are simulation results. All the results are obtained from the setting of an initial single defector. The network size is 2500. Each data point is obtained by averaging over 20 game and 10 networks realizations.

small-degree nodes connected to the hub become important. In particular, since most neighbors of the largest hub are agents with the smallest degree, their death induced by the defection of the hub can play the dominant role in the cascading process of elimination. The problem of predicting the critical value of α is equivalent to analyzing the stability of those smallest-degree nodes that are the direct neighbors of the hub. The critical value of α is then given by Eq. (11). We find that the smallest-degree agents cannot survive when they lose one interaction, so a cascading process of death or extinction can arise. This gives the condition $k'_{min} = k_{min} - 1$. We then have, from Eq. (11) and the property $\langle k \rangle = 2k_{min}$, for standard scale-free networks,[50]

$$\alpha_c = \frac{k_{min}}{k_{min}+1} = \frac{\langle k \rangle}{\langle k \rangle + 2}. \tag{28}$$

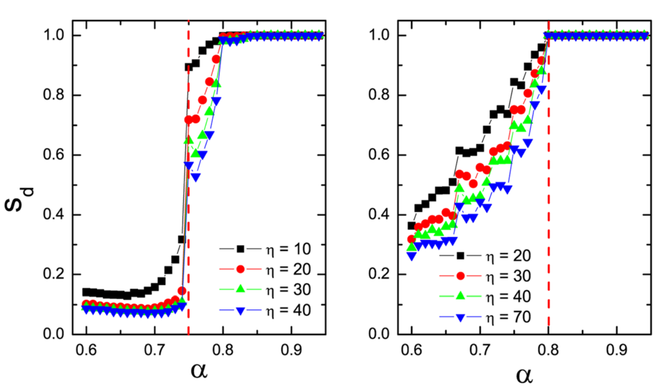

FIG. 16. (Color online) Dependence of the size of death, s_d, on the tolerance parameter α for different values of game parameter η in PGG with a single defector initially. The left panel is for the average degree $\langle k \rangle = 3$ and the right panel is for $\langle k \rangle = 4$. The red dashed lines are analytical predictions. Each data point is obtained by averaging over 500 realizations for 10 networks. The network size is 1000.

Simulation results are shown in Fig. 16 for two different values of the average degree $\langle k \rangle$. Again there is a good agreement between the predicted and numerical values of α_c.

For PDG and SG, α_c can be calculated as

$$\alpha_c = \frac{k_{min} - 1}{k_{min}} = \frac{\langle k \rangle - 2}{\langle k \rangle}, \tag{29}$$

which has also been verified numerically.

VI. CONCLUSIONS AND DISCUSSIONS

We have constructed a "minimal" physical model to investigate catastrophic behavior in networked systems governed by evolutionary games and have found two generic phenomena that do not depend on the system details such as the network topology and game types: (1) defection strategies for temporal high payoff can result in large-scale cascading failures or even the collapse of the entire system and (2) the optimal strategy for surviving catastrophic failures is cooperation. In particular, we uncover the emergence of clusters of cooperators after a large-scale cascading failure. Defection strategies, while being capable of generating high payoff in short time scales, can trigger a negative feedback mechanism that weakens the viability of defectors and leads to their ultimate death. In contrast, cooperation, while often temporally outperformed by defection, can survive eventually in the form of clusters that ensure enough profits for their members to resist deficit as well as the invasion of defectors in the long run. These results suggest that selfish and greedy strategies can be quite harmful for the health of the underlying networked system, be social or economical. In order to sustain the normal functioning of the system and to maximize individual agents' gain in the long run, cooperation is absolutely the optimal strategy. These results provide insights into, for example, the phenomenon of large-scale bankruptcy of financial institutions and corporations witnessed during the recent global economical recession.

Our model in fact describes the *co-evolution* of game dynamics and network topology. In particular, the dynamical evolution is triggered by selfish strategies that result in the death of both defectors and cooperators, thereby altering the network connecting structure from time to time. In contrast, in the absence of defectors, no agent dies, regardless of the tolerance and of the interaction patterns among agents.

Our model bears certain resemblance to the Susceptible-Infectious-Recovered (SIR) epidemic model if cooperators, defectors, and dead individuals are regarded as susceptible, infected, and recovered individuals, respectively, in the standard SIR setting. However, in our cascading model based on evolutionary-game dynamics, the strategy updating process is quite different from the propagating dynamics in the SIR model. Thus, that cascading failures are typical dynamical behaviors in our model does not necessarily imply that this type of catastrophic dynamics is also common in SIR epidemic models.

How to prevent the occurrence of the catastrophic behavior and to reduce damages is an issue of practical importance. Our model may be helpful to address this problem by incorporating other mechanisms, such as punishment or volunteering.[19] Exploring control strategies to suppress cascading processes in social and economical systems remain an open problem. The present model may shed new light and stimulate further efforts.

ACKNOWLEDGMENTS

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